The sonic flow about some symmetric half-bodies

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SUMMARY

The approximate Tricomi equation relevant to sonic speed of two-dimensional small-disturbance flow is solved by separation of variables, where these are certain stipulated functions of the Cartesian velocity perturbations. The symmetric flow patterns obtained from this solution are shown to correspond to those about half-bodies, whose ordinates vary as x^n where $0{\cdot}4 \leqslant n < 1$, and x is the distance along the plane of symmetry. The surface pressures on such bodies are deduced.

In particular, the body whose ordinates vary as $x^{2/5}$ has a sonic surface velocity, except at the nose, where an edge-force which causes a drag force is shown to exist. On the assumption that a free-stream breakaway (at sonic velocity) occurs at the shoulder of a body, this solution thereby yields the flow about an aerofoil of the same shape having a flat base. This bluff-nosed section has only a little more than half the drag of a wedge on the same chord and base.

INTRODUCTION

The use of the hodograph transformation of the equations of inviscid, transonic fluid flow is too well known to need description here, particularly in its approximate form which leads to the Tricomi equation. Whilst this is not the place for a rigorous criticism of the assumptions involved in this approximation, it will be recalled that its essential basis is that the fluid perturbation velocity is in general of small magnitude compared with the speed of sound. It is not unusual for this approximation to be used for a flow about a body with a stagnation point, though plainly the assumptions made break down in the immediate locality of such a point. Indeed the perturbation velocity becomes singular at this point. However, by reducing some relative proportion representing the body geometry and governing the magnitude of the disturbance created—such as the (thickness/chord) ratio of a wing section—it is possible to arrange that the perturbation of velocity exceeds any prescribed limit only within a region whose dimensions are small compared with those of the body.

In the present paper we shall discuss the flow about *bluff* bodies, for which there is inevitably a 'stagnation point' represented by a singularity in the perturbation velocity, whose form differs according to the shape of the body. For instance, the rise in speed along the body surface from the nose will be shown to be more rapid on bluff shapes than on those with more nearly sharp noses. However, we are also dealing with *half*-bodies, whose scale is characterized by a single length—such as, for example, the nose radius of the body if it happens to be parabolic. It transpires that the approximation is inapplicable within a region about the nose whose dimensions are of the order of this characteristic length. Thus the solution is only valid as applied to the flow at such distances from the nose (large compared with this length) that the perturbation velocities are bounded within prescribed small limits. In particular, the surface conditions are only correctly simulated where the surface slope is small. Relative to this scale, the region of failure is small, and by the same token the actual shape of the body surface in this region is really immaterial.

A precise distinction is drawn if it is stated that, by using the Tricomi equation, we can strictly deal with only the *asymptotic* behaviour (towards infinity) of the sonic flow about certain half-bodies, whose *asymptotic* shape is stipulated. Asymptotic conditions are approached within any desired accuracy at distances which are large compared with the characteristic length of the body asymptote (which we shall later identify by the symbol ϵ). Usually a solution is sought of this Tricomi equation by the method of separation of variables, these being in fact the Cartesian perturbation velocity components of the fluid motion. We seek here a solution of the equation by the same method, using, however, two different independent variables which are functions of both these velocity components. These particular functions have appeared before in various essays on the subject of transonic flow, though it does not appear that they have been used in the way employed here.

It may be that, by superposition of such solutions as those obtained below, it is possible to determine the flow about certain closed bodies; it is certain that various asymmetric flow fields about lifting surfaces may be derived in this way. These could be of practical interest, though the appearance of supersonic regions of flow and limit lines greatly complicates such analysis. However, we shall content ourselves here with the simple basic solutions which yield the flow about semi-infinite and symmetric bodies whose ordinates vary as x^n, where x is the distance downstream of the nose in the free-stream direction. Provided the power index n is between 0·4 and 1, the flow velocity is nowhere supersonic. The half-body whose ordinates vary as $x^{2/5}$ is of particular interest since, within the accuracy of the approximation, the surface speed is constant and sonic; because of this the flow may also be envisaged as that about a finite aerofoil of this nose shape having a flat base, with free-streamlines extending from the corners of its base to enclose the wake. It might be concluded that such an aerofoil has zero pressure drag, but this is not so because of the existence of an edge force at the nose; however, its drag is much less than that of a wedge on the same base and of the same chord. The parabola is another member of the family of half-bodies which is also of interest, since it coincides with the displacement surface of a laminar boundary layer on a flat plate. The semi-infinite wedge does not yield a bounded field of velocity and is not amenable to treatment.

In general, we treat only the surface conditions and not the complete streamline pattern, though computation of this would be possible. The hodograph solution is first obtained and then its interpretation in the physical plane is discussed. Finally, the particular applications already mentioned are amplified.

If a liberal view is taken, it can be suggested that the results lend support to the general belief that a bluff-nose can provide less drag than a sharp one at the speed of sound. The results may prove of use in design problems of both internal and external aerodynamics; and they enable the extent of the inviscid wake to be calculated in conditions of shock-free flow separation at sonic speeds.

The hodograph solution

Suppose that the velocity of flow of a gas is everywhere nearly sonic; then the Cartesian components U and V (with the direction of the free stream in the direction of the positive x-axis) can be written as

$$U = a^*\left(1-\frac{u}{\gamma+1}\right), \qquad V = \frac{v}{\gamma+1}a^*, \tag{1}$$

where u and v are small compared with unity, and a^* is the critical speed of sound. Using u and v as independent variables, it can then be shown that the stream function Ψ and velocity potential Φ of the flow obey the equations

$$\Phi_u = u\Psi_v, \qquad \Phi_v = -\Psi_u.$$

The second of these relations is satisfied by defining a function $\Omega(u, v)$ such that

$$\Phi = -\Omega_u, \qquad \Psi = \Omega_v, \tag{2}$$

and the first then yields the well-known Tricomi equation

$$\Omega_{uu}+u\Omega_{vv} = 0. \tag{3}$$

Solutions of this equation by means of separation of the variables u and v have frequently been found of interest. We here employ the same means, but we separate instead newly introduced independent variables,

$$w = \tfrac{4}{9}u^3+v^2, \qquad \xi = v^2/w = 1-\tfrac{4}{9}(u^3/w). \tag{4}$$

We shall concern ourselves only with flows without supersonic regions so that $u \geqslant 0$; hence $w \geqslant 0$, and $0 \leqslant \xi \leqslant 1$. If supersonic regions were present the value of ξ would not lie in this range. Positive and negative values of v are distinguished by the sign of $\xi^{1/2}$. Using these new variables, equation (3) becomes

$$w^2\Omega_{ww}+\tfrac{7}{6}w\Omega_w+(\tfrac{1}{2}-\tfrac{7}{6}\xi)\Omega_\xi+\xi(1-\xi)\Omega_{\xi\xi} = 0. \tag{5}$$

If we seek a solution of the form

$$\Omega = f(w)g(\xi), \tag{6}$$

then the functions f and g satisfy the equations

$$\left.\begin{aligned} w^2f''+\tfrac{7}{6}wf'+(\tfrac{1}{144}-\mu^2)f=0,\\ \xi(1-\xi)g''+(\tfrac{1}{2}-\tfrac{7}{6}\xi)g'-(\tfrac{1}{144}-\mu^2)g=0,\end{aligned}\right\} \tag{7}$$

where, for convenience, $(\frac{1}{144}-\mu^2)$ has been chosen as the constant of separation.

The first of these equations may be solved to give

$$f(w)=(C_1w^{\mu}+C_2w^{-\mu})w^{-1/12}, \tag{8}$$

where C_1 and C_2 are disposable constants. The second may be reduced to Legendre's equation, although its solution in the range $\xi<1$ may be more conveniently expressed formally in terms of hypergeometric functions,

$$g(\xi)=B_1F(\tfrac{1}{12}+\mu,\tfrac{1}{12}-\mu;\tfrac{1}{2};\xi)+B_2\xi^{1/2}F(\tfrac{7}{12}+\mu,\tfrac{7}{12}-\mu;\tfrac{1}{2};\xi), \tag{9}$$

where again B_1, B_2 are adjustable constants.

Since the hypergeometric functions are not affected by the sign of μ, it is no loss of generality to put, say, $C_2=0$ in (8), and take the value of C_1 as unity. Further, we shall restrict the analysis to flows symmetric about the free-stream direction so that Ψ is an odd function, and Φ and Ω are even functions, of v (i.e. of $\xi^{1/2}$). Thus we take $B_2=0$ and it follows on substituting from (8) and (9) in (6) that a typical solution of the form required is

$$\Omega=B_1w^{\mu-1/12}F(\tfrac{1}{12}+\mu,\tfrac{1}{12}-\mu;\tfrac{1}{2};\xi). \tag{10}$$

Using (2) and (4), together with (10), we obtain after some rearrangement

$$\left.\begin{aligned} \frac{1}{a^*l}\Phi&=-\tfrac{2}{3}u^2w^{-m}F(m,\tfrac{7}{6}-m;\tfrac{1}{2};\xi)\\ &=-(\tfrac{3}{2})^{1/3}w^{-m+2/3}(1-\xi)^{2/3}F(m,\tfrac{7}{6}-m;\tfrac{1}{2};\xi),\\ \frac{1}{a^*l}\Psi&=2(m-\tfrac{2}{3})vw^{-m}F(m,\tfrac{7}{6}-m;\tfrac{3}{2};\xi)\\ &=2(m-\tfrac{2}{3})w^{-m+1/2}\xi^{1/2}F(m,\tfrac{7}{6}-m;\tfrac{3}{2};\xi),\end{aligned}\right\} \tag{11}$$

where, for convenience, we have put $m=\frac{13}{12}-\mu$, and replaced $2(\mu-\frac{1}{12})B_1$ by $a^*l>0$, say.

THE INTERPRETATION OF THE FLOW IN THE PHYSICAL PLANE

To interpret the flow pattern in the physical plane determined by the solution (11), we need to invoke the following known property of the hypergeometric functions, which are of course equal to unity at $\xi=0$ and are bounded and continuous in the range $0\leqslant\xi\leqslant1$, except possibly at $\xi=1$. Provided that $m\geqslant\frac{3}{2}$, there exist numbers ξ_1, ξ_2 where $0<\xi_1<\xi_2\leqslant1$, such that $F(m,\frac{7}{6}-m;\frac{3}{2};\xi)>0$ for $0<\xi<\xi_2$, and vanishes at $\xi=\xi_2$; and such that $F(m,\frac{7}{6}-m;\frac{1}{2};\xi)$ changes sign just once in $0\leqslant\xi<\xi_2$ at $\xi=\xi_1$. If $m<\frac{3}{2}$ it may be remarked, in passing, that supersonic regions of the flow and limit lines make their appearance.

Interpreting the powers of w by their positive real values (and supposing $m\geqslant\frac{3}{2}$) we see that at infinity (where Φ and Ψ are infinite) we have $w=0$

(and ξ arbitrary), so that from (1) and (4) the velocity has there decayed to that of the free stream. Further, the point $\Phi = \Psi = 0$ corresponds to a singularity in velocity ($w = \infty$ with ξ arbitrary). We see that Ψ, but not Φ, changes sign with v (and $\xi^{1/2}$) as required earlier to meet the provisions of flow symmetry, so that we may restrict ourselves to consider the upper half-plane where $\Psi \geqslant 0$. For all negative Φ, this region is bounded by the line $\Psi = 0$ on which $\xi = 0$ but w takes all (non-negative) values, that is, on which u is positive but $v = 0$. Evidently this is a 'stagnation' streamline, the 'stagnation' point at $\Phi = \Psi = 0$ being a singular point of the velocity distribution, as is the usual consequence of the approximations involved. The line $\xi = \xi_1$ then maps point by point on to the equipotential line $\Phi = 0$ extending from the singularity, the region $0 < \xi < \xi_1$ for all non-negative w transforming in a one-to-one relationship to all points within the region upstream of this line. Finally, we note that the line $\xi = \xi_2$ defines the continuation of the stagnation streamline $\Psi = 0$ with Φ taking all positive values, the region $\xi_1 < \xi < \xi_2$ mapping on to that downstream of $\Phi = 0$. Since ξ, and so also v, $\neq 0$ on $\Psi = 0$ downstream of the stagnation point, the stagnation streamline divides at $\Phi = 0$ into two branches (symmetric about the upstream stagnation streamline), the value of v being equal and opposite on each side. We see that the flow is a symmetric one about a half-body extending downstream of $\Psi = \Phi = 0$ with no singularities in the flow external to it.

The shape of the half-body in the physical plane will now be determined. In so doing, we will suppose that the value of $\xi_2 \equiv \xi_2(m)$ has been found (as already defined) to be the least positive real root of

$$F(m, \tfrac{7}{6}-m;\, \tfrac{3}{2};\, \xi_2) = 0, \qquad \text{for} \quad m \geqslant \tfrac{3}{2}; \tag{12}$$

for brevity it is convenient to put

$$(\tfrac{3}{2})^{1/3}(1-\xi_2)^{2/3} F(m, \tfrac{7}{6}-m;\, \tfrac{1}{2};\, \xi_2) = -\delta/l, \quad \text{say}, \tag{13}$$

where δ replaces l as an arbitrary positive length, $(\delta/l) > 0$, and is bounded for any $m \geqslant \frac{3}{2}$. We see then from (4) and (11) that on the upper surface of the half-body ($\Psi = 0$, $\Phi \geqslant 0$, $v \geqslant 0$)

$$\left.\begin{aligned} u &= [\tfrac{9}{4}(1-\xi_2)w]^{1/3}, \\ v &= (\xi_2 w)^{1/2}, \\ w &= \{\Phi/(a^*\delta)\}^{-3/(3m-2)}, \end{aligned}\right\} \tag{14}$$

all powers having the positive real interpretation. To identify the velocity field with the Cartesian coordinates (x, y) of the physical plane, we note that on a streamline changes of position coordinate are related to changes in Φ, within the accuracy of the approximation used in deriving the basic equations (2) and (3), by the expressions

$$dx = d(\Phi/a^*), \qquad dy = \frac{\quad}{\gamma+1} d(\Phi/a^*). \tag{15}$$

If the stagnation point $\Phi = \Psi = 0$ is the origin of the Cartesian axes, it follows on substituting from (14) in (15) that on the upper body surface ($\Psi = 0$)

$$\frac{dy}{dx} = \frac{\xi_2^{1/2}}{\gamma+1}\left(\frac{\delta}{x}\right)^{3/(6m-4)};$$

whence on integration, its shape is given by

$$\left.\begin{aligned}(y/\epsilon) &= (x/\epsilon)^n,\\ \text{where } n = \frac{6m-7}{6m-4} \quad &\text{and } \epsilon = \left[\frac{\xi_2^{1/2}}{n(\gamma+1)}\right]^{1/(1-n)}\delta.\end{aligned}\right\} \tag{16}$$

Since $m \geqslant \frac{3}{2}$ it follows that the index n can take any value between 0·4 and unity.

The pressure coefficient C_P is related to the velocity changes with sufficient accuracy by the equation

$$C_P = \frac{2}{\gamma+1}u, \tag{17}$$

whence from (14), (15) and (16) it follows that on the body surface

$$\left.\begin{aligned}C_P &= \alpha(n)(dy/dx)^{2/3} = n^{2/3}\alpha(n)(\epsilon/x)^{2(1-n)/3},\\ \text{where} \qquad \alpha(n) &= \left[\frac{18}{\gamma+1}\left(\frac{1-\xi_2}{\xi_2}\right)\right]^{1/3}.\end{aligned}\right\} \tag{18}$$

The function $\alpha(n)$ may be determined by solving (12); this demands a knowledge of the behaviour of the quoted hypergeometric function, which may, however, be related to evaluated transcendental functions if the value of m is an integral multiple of $\frac{1}{6}$. In particular, we note that for $m \to \frac{3}{2}+0$

$$\left.\begin{aligned}&n \to \frac{2}{5}+0, \quad \xi_2 \to 1-0, \quad \alpha(n) \sim \frac{5}{18}\left(\frac{18}{\gamma+1}\right)^{1/3} B\left(\frac{1}{3},\frac{3}{2}\right)(5n-2),\\ &\text{and for } m \to +\infty\\ &n \to 1-0, \quad \xi_2 \sim \frac{\pi^2}{4m^2}, \quad \alpha(n) \sim \left[\frac{72m^2}{\pi^2(\gamma+1)}\right]^{1/3},\end{aligned}\right\} \tag{19}$$

where $B(a,b)$ is the beta function.

Thus α varies from 0 to ∞ as n varies from 0·4 to 1. Its variation is sketched in figure 1, composed from a discrete number of solutions of (12). The expressions for the hypergeometric function appearing in (12) with values of $6m$ between 4 and 9 are summarized for convenience in table 1. The expressions with larger integral values of $6m$ both for this and also for the other hypergeometric function of the fundamental solution (11), may be determined from the tabulated functions by elementary operations. The manner of these operations is indicated in table 1.

The point made in the Introduction about the breakdown of the theory close of the stagnation point is best appreciated by referring to equation (18), from which it will be seen that C_P and dy/dx are in general finite on the surface for finite (x/ϵ), where (from equation (16)) it will be observed that (y/ϵ) is also finite. In general the velocity perturbations will only be small, as required by the Tricomi approximation, at distances from the origin which are large compared with ϵ. Thus if it is required that C_P (or u) has an

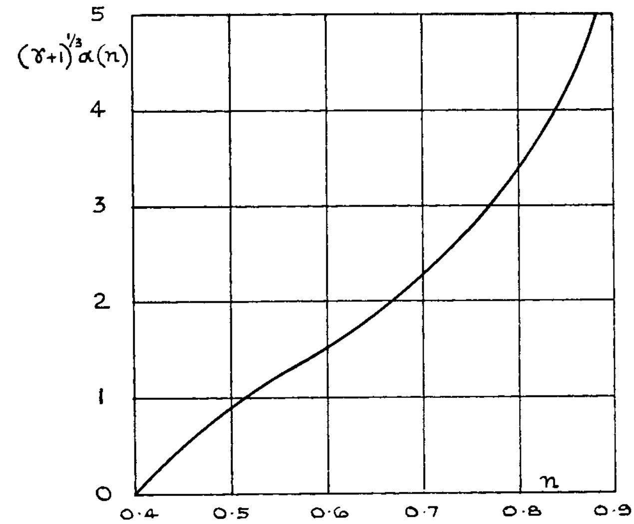

Figure 1. Variation of the function $\alpha(n)$ introduced in equation (18).

$m = \frac{2}{3}$	$F(\frac{2}{3}, \frac{1}{2};\ \frac{3}{2};\ \xi) = \frac{1}{2}\xi^{-1/2}B_\xi(\frac{1}{2},\frac{1}{3})$
$m = \frac{5}{6}$	$F(\frac{5}{6}, \frac{1}{3};\ \frac{3}{2};\ \xi) = \frac{3}{2}\xi^{-1/2}[(1+\xi^{1/2})^{1/3}-(1-\xi^{1/2})^{1/3}]$
$m = 1$	$F(1, \frac{1}{6};\ \frac{3}{2};\ \xi) = \frac{1}{2}\xi^{-1/2}(1-\xi)^{1/3}B_\xi(\frac{1}{2}, -\frac{1}{3})$
$m = \frac{7}{6}$	$F(\frac{7}{6}, 0;\ \frac{3}{2};\ \xi) = 1$
$m = \frac{4}{3}$	$F(\frac{4}{3}, -\frac{1}{6};\ \frac{3}{2};\ \xi) = \frac{3}{16}\xi^{-1/2}[(3\xi^{1/2}-1)(1+\xi^{1/2})^{1/3}+(3\xi^{1/2}+1)(1-\xi^{1/2})^{1/3}]$
$m = \frac{3}{2}$	$F(\frac{3}{2}, -\frac{1}{3};\ \frac{3}{2};\ \xi) = (1-\xi)^{1/3}$
$m = a+1$	$F(a+1, \frac{7}{6}-a-1;\ \frac{3}{2};\ \xi) = \frac{(2a-1)(6\xi+6a-1)}{4a(3a+1)} F(a, \frac{7}{6}-a;\ \frac{3}{2};\ \xi) + \frac{(12a-1)}{4a(3a+1)} F(a, \frac{7}{6}-a;\ \frac{1}{2};\ \xi)$, where $F(a, \frac{7}{6}-a;\ \frac{1}{2};\ \xi) = 2\xi^{1/2}\frac{d}{d\xi}[\xi^{1/2}F(a, \frac{7}{6}-a;\ \frac{3}{2};\ \xi)]$

Table 1. Closed expressions for the hypergeometric functions $F(m, \frac{7}{6}-m;\ \frac{3}{2};\ \xi)$. The expression $B_\xi(a, b) = \int_0^\xi t^{a-1}(1-t)^{b-1}\,dt$ is the incomplete beta-function.

order of magnitude $(1/N)$, then v has a smaller order of magnitude $(1/N^{3/2})$, and it may be shown that this applies in general where

$$\frac{\epsilon}{x} = O\left(\frac{1}{N^{3/(2-2n)}}\right) \quad \text{or} \quad \frac{\epsilon}{y} = O\left(\frac{1}{N^{(4-n)/(2-2n)}}\right).$$

The singularity representing the stagnation point becomes more intense as the degree of bluffness, represented by the exponent $(1-n)$, increases. Thus equation (18) shows that as n is reduced towards 0·4 the surface speed recovers more rapidly from the 'stagnation point', until in the limit, on the body shape given by $n = 0{\cdot}4$ it jumps discontinuously from infinity to the sonic speed. Some typical examples are illustrated by figure 2.

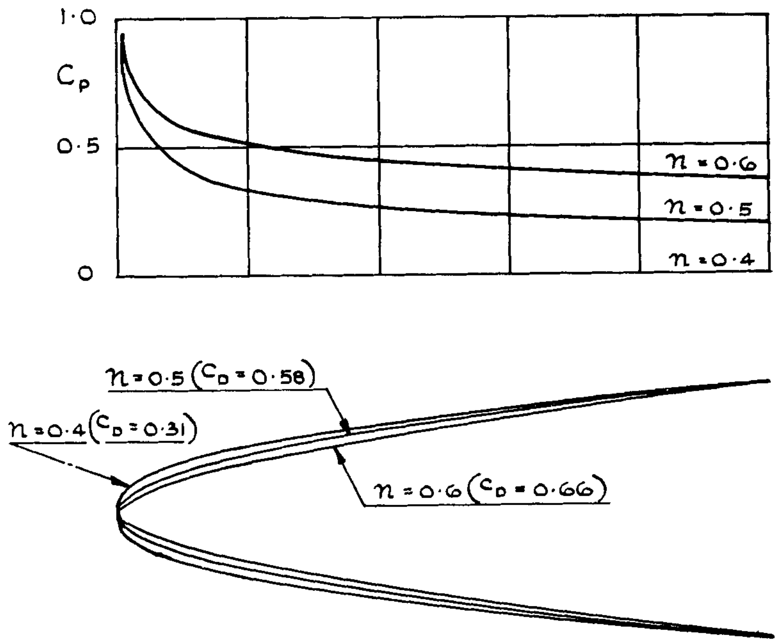

Figure 2. Comparison of nose-shapes on some of the half-bodies, together with the corresponding pressure distributions and drag coefficients on the frontal area (over portions shown). Scaling the ordinates by a factor $(1/N)$ reduces the pressure and drag coefficients in the ratio $(1/N)^{2/3}$.

Some particular solutions of interest

The simplest case $n = 0{\cdot}4$ (or $m = \frac{3}{2}$) is of particular interest, since it will be seen from (16) and (18) that it yields the flow past a half-body (whose ordinates vary as $x^{2/5}$) over the whole of whose surface the velocity is sonic and the pressure equal to that of the free-stream. It might therefore be thought that the drag on the body is zero, but an 'edge force' exists at the stagnation point, as may in fact be demonstrated by considering the drag derived from the pressure distribution along any complete streamline other than $\Psi = 0$. It is more simple to show this, however, by noting that the drag on that part of any of the bodies defined by (16) ahead of the plane $x =$ const., where the pressure coefficient is C_P, can be expressed by means of a coefficient C_D based on the local frontal area and given by

$$C_D = \frac{3n}{5n-2}C_P. \tag{20}$$

Thus in particular, allowing $n \to 0{\cdot}4$ from above, we find from (20) with the help of (16) and (19), that the drag of the half-body $y = \epsilon^{3/5}x^{2/5}$ ahead of the plane $x = \text{const.} > 0$ is

$$\tfrac{4}{5}(\tfrac{5}{3})^{1/3}B(\tfrac{1}{3},\tfrac{3}{2})\tfrac{1}{2}\rho^* a^{*2}\epsilon(\gamma+1)^{-1/3}. \tag{21}$$

This is independent of x and so the drag originates from the nose. The type of singularity in the hodograph which yields such an edge force is found by placing $m = \frac{3}{2}$ in (11).

This appearance of an edge force is interesting, and it seems to be the first occasion that it has been noticed in the study of transonic flow. A finite force associated with a singularity is known to appear at a radiused leading edge in the treatment of the flow about it by linearized theory, provided only that the free-stream velocity component normal to the edge is subsonic. Although rounded leading edges are strictly outside the scope of linearized theory, it is further known that nevertheless the forces are correctly given. It is not possible to draw the same conclusion in the present instance, because there is no more exact treatment available of particular cases by which the issue may be judged. The most that can be said with assurance is that a body whose shape is asymptotic to a curve $y = \epsilon^{3/5}x^{2/5}$ has a drag only by virtue of the pressure distribution over the surface at distances of order ϵ from the nose.

If the hypothesis is adopted that a free-streamline extends from the shoulder of a flat-based aerofoil, enclosing fluid at the free-stream pressure (so that there is no base-drag), we see further that this same solution (with $n = 0{\cdot}4$) then yields the sonic flow about a finite aerofoil (whose ordinates $y \propto x^{2/5}$) with the free-streamline forming a continuation of the surface downstream of its flat base. The ultimate width of the 'wake' contained within the free-streamlines would be infinite (at infinity downstream), which is the same conclusion that is reached in examining the flow about a flat-based wedge. The drag coefficient (on frontal area) of the curved aerofoil thus deduced is, from (16) and (21),

$$C_D = 1{\cdot}20(y_{\max}/c)^{2/3}(\gamma+1)^{-1/3}$$

where c is the chord and $y_{\max}$ the semi-ordinate at the base. For a wedge, the numerical factor is 1·89, instead of 1·20 (Helliwell & Mackie 1957).

If this same hypothesis about the wake is made in dealing with flow about an arbitrary aerofoil, with separation at the point of maximum thickness, we see that the shape of the free-streamline representing the wake boundary will have ordinates asymptotic to $y = \epsilon^{3/5}x^{2/5}$. Further, from a consideration of the flux of momentum at infinity, it will be evident that the characteristic length ϵ of this asymptote is determined by the drag of the aerofoil in accordance with the expression (21). These deductions are in fact confirmed by the work on the wedge previously mentioned.

The half-body given by $y \propto x^{1/2}$ is also of some interest as, on account of its parabolic shape, it evidently shows the displacement effect at sonic speed of a laminar boundary layer on a semi-infinite flat plate aligned with the stream direction. The self-induced pressure field of the boundary layer is then seen from (18) to vary as $1/x^{1/3}$. The theory breaks down when applied to the semi-infinite wedge ($n = 1$) since it predicts an unbounded surface velocity. This is perfectly compatible with results for the finite wedge, because these show that the surface velocity becomes infinite at the nose; the semi-infinite wedge, on the other hand, has no characteristic length, and so the surface speed must be constant, with a value corresponding to that at the nose of the finite wedge.

Reference

Helliwell, J. B. & Mackie, A. G. 1957 *J. Fluid Mech.* **3**, 93.